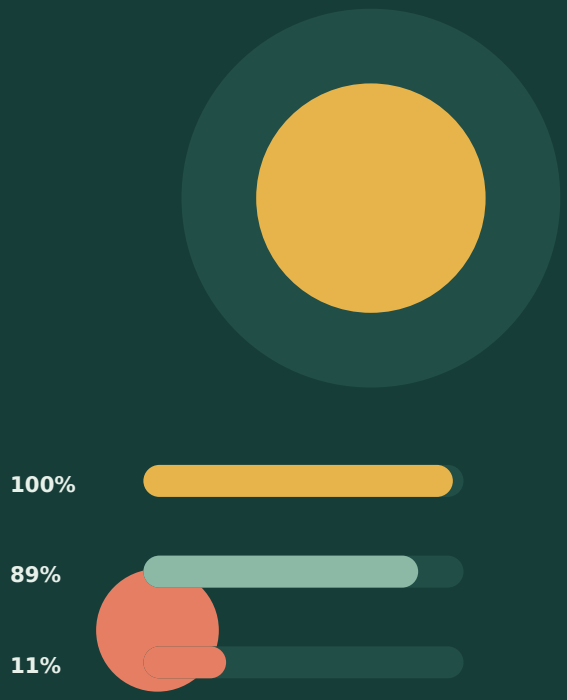
THE

ALLAIS PARADOX



Why can a sure outcome change the way we choose under risk?

THE KEY IDEA

Two pairs of lotteries can reveal a tension between what people choose and what expected utility theory requires.

A B C D

A visual guide to certainty, probability, and the independence axiom

Choose one option in each pair

Imagine that each option is a lottery. Make one choice in Pair 1 and one choice in Pair 2. The amounts below are illustrative money prizes.

PAIR 1 — **A certain prize versus a high-upside lottery**

A — The sure option

100%	$1 million

B — The risky option

10%	$5 million
89%	$1 million
1%	$0

PAIR 2 — **No guaranteed prize: compare the upside**

C — The safer option

11%	$1 million
89%	$0

D — The high-upside option

10%	$5 million
90%	$0

THE FAMILIAR RESPONSE PATTERN

A in Pair 1 **THEN** **D** in Pair 2

This A-then-D pattern is the classic Allais-type preference reversal.

The common consequence

The two pairs differ in one important way: Pair 1 contains a shared 89% chance of receiving $1 million. Independence says that adding the same consequence to both options should not change their ranking.

PAIR 1

A **100% → $1m**

The sure $1 million can be split into:

89% × $1m | 11% × $1m

B **The risky lottery**

Its 89% branch is also $1 million:

89% × $1m | 11% × risk

THE COMMON CONSEQUENCE

Both A and B give the same 89% chance of $1 million. Set that identical branch aside.

WHAT REMAINS AFTER THE SHARED 89% BRANCH?

A: 11% part

11% chance of $1m

B: 11% part

10% chance of $5m

1% chance of $0

PAIR 2 IS THE SAME RESIDUAL COMPARISON

C offers 11% for $1m; D offers 10% for $5m and 90% for $0. The shared 89% zero branch does not change the comparison under independence.

So, if you rank A above B, expected utility theory requires C above D as well.

What expected utility requires

The paradox is not simply that people ignore expected money. It is a conflict with the independence axiom of von Neumann–Morgenstern expected utility theory.

SUPPOSE A IS PREFERRED TO B

$$u(1) > 0.10u(5) + 0.89u(1) + 0.01u(0)$$

Subtract the common 0.89u(1) term:

$$0.11u(1) > 0.10u(5) + 0.01u(0)$$

Add 0.89u(0) to both sides:

$$0.11u(1) + 0.89u(0) >$$
$$0.10u(5) + 0.90u(0)$$

THEREFORE: C > D

DO NOT CONFUSE THESE

Expected value

Average monetary payoff if the lottery were repeated many times.

Expected utility

Probability-weighted utility; it allows people to dislike risk.

EXPECTED MONETARY VALUE (IN MILLIONS)

A	B	C	D
$1.00m	$1.39m	$0.11m	$0.50m

A and D is not the expected-value-maximizing pattern. The key result is that it cannot be explained by one consistent von Neumann–Morgenstern expected-utility ranking.

Turn the paradox into a classroom study

A short choice task can reveal how certainty, probability, and framing shape risky decisions. Use the same wording and payment rule for every participant.

1

Present Pair 1

Ask participants to choose A or B. Do not explain the paradox before the choice.

2

Present Pair 2

Ask for C or D. Counterbalance pair order across participants if order effects matter.

3

Record the pair

Keep the joint response: A–C, A–D, B–C, or B–D. The classic pattern is A–D.

4

Debrief the logic

Show the common 89% branch and explain why independence links the two choices.

HOW TO INTERPRET A–D

It is evidence of a choice pattern that violates independence. It is not, by itself, proof that a participant is irrational or that one psychological mechanism explains every response.

ASK PARTICIPANTS

Did certainty matter? Did you mentally cancel the shared outcome? Did the larger prize attract your attention?

DESIGN CAREFULLY

Check that probabilities sum to 100%. Explain incentives and payment selection clearly. Separate misunderstanding from genuine preference.

FURTHER READING Allais (1953), Econometrica, 21(4), 503–546. • Kahneman & Tversky (1979), Econometrica, 47(2), 263–291. • Slovic & Tversky (1974), Behavioral Science, 19, 368–373.